Optimum Reception in an Impulsive Interference Environment—Part II: Incoherent Reception

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Abstract—In Part I, the relevant statistical properties of the recently developed statistical-physical model of generalized impulsive interference have been briefly reviewed (for sub Class A noise) and then applied specifically to optimum coherent detection. It is shown that by using optimum and (locally optimum) detection algorithms (canonically and explicitly derived), substantial savings in signal power and/or spectrum space can be achieved for operation in these highly non-Gaussian interference environments. This paper (Part II) extends the preceding analysis to cover various important cases of incoherent reception. The same general model for narrow-band (Class A) impulsive interference and interference examples used in Part I are again employed here. In addition to providing both canonical LOBD structures and expressions for performance, this permits explicit quantitative comparisons between coherent and incoherent reception for common classes of specific digital signal waveforms.

I. INTRODUCTION

As WE have already noted (Part I), man-made electromagnetic interference (or noise) has become a problem of increasing concern to the telecommunications community, particularly in the face of increasing demands on available bandwidth resources. The man-made EM environment, and much of the natural one as well, are basically "impulsive"; i.e., have a highly structured character, with noticeable probabilities of large interference levels, unlike the normal (Gaussian) noise processes usually assumed (and extensively analyzed in earlier work). In the previous paper (Part I), we have described the relevant statistical properties of a recently developed canonical, statistical-physical model of impulsive interference [1, 2], and we have then used this model specifically to obtain the associated optimum detection algorithms and to analyze detector performance for coherent binary signals in Class A interference as well as to obtain general structures and performance canonically. Class A interference arises from sources whose emission spectra are comparable to or narrower than the bandpass of the receiver in use. [For a comprehensive description of Class A noise, and the noise model generally, which includes Class B interference [i.e., "broad-band noise"], and combinations of Class A and Class B types, see Ref. [2].] In this paper (Part II) we treat the incoherent (i.e., unknown signal phase) binary detection case. In general, incoherent detection problems are technically much more difficult to treat than coherent ones, and the cases considered here are no exception to this observation. The basic binary, incoherent detection situation here may be concisely stated in the usual way [3] as a composite two-hypothesis statistical test:

$$\left.\begin{aligned} H_1&: X(t) = S_1(t, \theta) + Z(t), \quad 0 \leq t < T \\ H_2&: X(t) = S_2(t, \theta) + Z(t), \quad 0 \leq t < T \end{aligned}\right\}, \tag{1}$$

where Z is the accompanying interference and the vector θ denotes the unknown parameters of the signals. These unknown parameters may be phase, amplitude, frequency, or any combination of them. The optimum detector is well known to be a generalized likelihood ratio $\Lambda(X)$, with decisions made vis-a-vis a threshold K, viz.,

$$\Lambda(X) = \frac{\int_\theta p(X \mid H_2) p(\theta)\, d\theta}{\int_\theta p(X \mid H_1) p(\theta)\, d\theta} = \frac{p_2(X)}{p_1(X)} \underset{H_2}{\overset{H_1}{\lessgtr}} \frac{q_1 K}{q_2}. \quad \text{(decide:)} \tag{2}$$

As before, $p(X \mid H_{1,2})$ is given by (32) of Part I, with $p(X \mid H_{1,2}) = p_Z(X - S_{1,2}(\theta))$ (e.g., (15) of Part I), and $p(\theta)$ denotes the pdf of our unknown parameters θ. Since our $p(X \mid H_{1,2})$ are given here for independent sampling by the Nth product of an infinite summation (see (15) of Part I), it is unlikely that the required averages (over θ) can be performed explicitly for general $p(\theta)$. [Later, however, we will show how, in special circumstances, the averages can be performed directly, reducing the problem of evaluation to one equivalent to the coherent case, so that the methods of the previous paper [Section III] can be used.]

In general, the chief new results here are: (i), various canonical structures for binary optimum incoherent threshold detection, particularly involving nonuniform phase distribution, and (ii), calculation of performances under Class A interference, including comparisons with conventional correlation receivers. Thus, we begin (in Section II) with a canonical (vis-a-vis the noise) treatment of the important case of incoherent threshold signals and consider the case of unknown amplitude and phase. For the unknown amplitude cases, we treat both "fast" and "slow" fading. In Section II we also consider the important case where phase estimation is used, with the ON-OFF incoherent case as a special situation of particular interest. In Section III, we extend the analysis to include general incoherent signals (without small-signal assumptions). Performance of the optimum (and locally optimum (LOBD)) incoherent detectors is next computed in Section IV, as is the

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performance (error probabilities) of the suboptimum systems (i.e., matched filter detectors, optimum for Gauss). These performance results are then compared for the same signaling situations. Optimum incoherent and optimum coherent receiver performance are similarly compared. Finally, in Section V we summarize the principal features of the many new results [obtained both in (Part I) and in this paper], along with a short discussion of their implications and further technical problems to be investigated for these highly non-Gaussian interference environments.

II. INCOHERENT DETECTION OF THRESHOLD SIGNALS

In this section we treat the case of the threshold signal or locally optimum Bayes detector (LOBD) for some representative incoherent reception situations:

A. Unknown Amplitude and Phase

Consider the standard (as received) fading, incoherent frequency-shift keying signals, with unknown amplitude and phase:

$$\left.\begin{aligned} S_1(t,\theta) &= a\cos(\omega_1 t + \phi) \\ S_2(t,\theta) &= a\cos(\omega_2 t + \phi) \end{aligned}\right\} \tag{3}$$

where a denotes the unknown amplitude and ϕ the unknown phase; a and ϕ we postulate to be independent, and in (2) above $\theta = \{a, \phi\}$. As before (Part I), we let the SNR be S; i.e., we choose $\langle a^2\rangle = 2S$. The likelihood ratio (2) for these discrete sampling cases is now written explicitly

$$\Lambda(X) = \frac{p_2(X)}{p_1(X)} = \frac{\int_\phi \int_a p_Z(X - S_2)p(\phi)p(a)\,d\phi\,da}{\int_\phi \int_a p_Z(X - S_1)p(\phi)p(a)\,d\phi\,da} \;\; \underset{H_2}{\overset{\text{decide: } H_1}{\lessgtr}} \; K, \tag{4}$$

where $p(a)$ and $p(\phi)$ are the pdf's of a and ϕ, and $s_{2i} = a\cos(\omega_2 t_i + \phi), i = 1, \cdots, N$, etc.

We start with the small signal case and as before use the Taylor expansion (33a) about ($S = 0$), now retaining terms at least $0(S^2)$ in the development. We next let the phase ϕ be uniformly distributed $(-\pi, \pi)$ and let the amplitude a have an arbitrary fading distribution. We consider two fading situations: 1, *slow fading*, and 2, *fast fading:*

Case 1. Slow Fading

First consider the slow fading case, where a is random, but constant over the detection period T. Then we have at once from (3)

$$\langle s_{2i}\rangle_\phi = \frac{a}{2\pi}\int_{-\pi}^{\pi} \cos(\omega_2 t_i + \phi)\,d\phi = 0 \tag{5a}$$

and

$$\langle s_{2i}s_{2k}\rangle_{\phi,a} = \frac{\langle a^2\rangle}{2}\cos(\omega_2 t_i - \omega_2 t_k). \tag{5b}$$

Now, dividing the numerator and denominator of $\Lambda(X)$, (4), by $p_Z(X)$ and assuming again N independent samples [cf. remarks in latter part of Section II, Part I], we obtain

$$\begin{aligned} \frac{p_2(X)}{p_Z(X)} &\approx 1 + \frac{1}{2p_Z(X)}\sum_{i=1}^N \sum_{k=1}^N \frac{\partial^2 p_Z(X)}{\partial x_i \partial x_k}\frac{\langle a^2\rangle}{2}\cos(\omega_2 t_i - \omega_2 t_k) \\ &\approx 1 + \frac{\langle a^2\rangle}{4}\sum_{\substack{i=1 \\ i\neq k}}^N \sum_{k=1}^N l(x_i)l(x_k)\cos(\omega_2 t_i - \omega_2 t_k) \\ &\quad + \frac{\langle a^2\rangle}{4}\sum_{i=1}^N \frac{d^2 p_Z(x_i)}{dx_i^2}\Big/ p_Z(x_i), \end{aligned} \tag{6}$$

with corresponding results for $p_1(X)$, where $l(x_i)$ denotes the nonlinearity obtained previously [cf. (35), Part I], e.g.,

$$l(x_i) \equiv \frac{d}{dx_i}\ln p_Z(x_i). \tag{7}$$

Since

$$\sum_{i=1}^N \frac{p_Z''(x_i)}{p_Z(x_i)} = \sum_{i=1}^N l'(x_i) + \sum_{i=1}^N [l(x_i)]^2, \tag{8}$$

we obtain

$$\begin{aligned} \frac{p_2(X)}{p_Z(X)} &\approx 1 + \frac{\langle a^2\rangle}{4}\sum_{i=1}^N \sum_{k=1}^N l(x_i)l(x_k) \\ &\quad \cdot \cos(\omega_2 t_i - \omega_2 t_k) + \frac{\langle a^2\rangle}{4}\sum_{i=1}^N l'(x_i). \end{aligned} \tag{9}$$

Therefore, since $\langle a^2\rangle/2 = S$, our desired test (4) is given by

$$\begin{aligned} &\frac{S}{2}\sum_{i=1}^N \sum_{k=1}^N l(x_i)l(x_k)\cos(\omega_2 t_i - \omega_2 t_k) \\ &\quad + (1-K) + \frac{S}{2}(1-K)\sum_{i=1}^N l'(x_i) \;\; \underset{H_2}{\overset{\text{decide: } H_1}{\lessgtr}} \;\; \frac{S}{2}K\sum_{i=1}^N \sum_{k=1}^N l(x_i) \\ &\quad \cdot l(x_k)\cos(\omega_1 t_i - \omega_1 t_k). \end{aligned} \tag{10}$$

We see that, for arbitrary thresholds, the receiver depends only on the average signal power S and is independent of the particular slow fading distribution. For the symmetrical case ($K = 1$), the receiver structure is also independent of S. Using a trigonometric identity for the $\cos(\omega_2 t_i - \omega_2 t_k)$ term, we obtain the following receiver ($K = 1$):

$$\begin{aligned} &\left[\sum_{i=1}^N l(x_i)\cos\omega_2 t_i\right]^2 + \left[\sum_{i=1}^N l(x_i)\sin\omega_2 t_i\right]^2 \;\; \underset{H_2}{\overset{\text{decide: } H_1}{\lessgtr}} \\ &\left[\sum_{i=1}^N l(x_i)\cos\omega_1 t_i\right]^2 + \left[\sum_{i=1}^N l(x_i)\sin\omega_1 t_i\right]^2, \end{aligned} \tag{11}$$

which is shown in Figure 1. We see that this is simply the standard noncoherent binary receiver optimum when the interference is Gaussian noise, now preceded by the nonlinear adaptive filter $l(x)$, (7). This result corresponds to that outlined earlier for coherent signals [cf. (35), Fig. 12, Part I]. If the threshold is other than 1, then the receiver also depends on the average signal power S and via (10), must contain a branch involving $l'(x)$, the derivative of the nonlinearity (7).

Case 2. Fast Fading

We next consider the fast-fading signal situation where $a \to \{a_i\}$ in (3). Then, we obtain (6), with $\langle a^2 \rangle$ replaced by $\langle a_i a_k \rangle$ therein, which, if a_i and a_k are independent, yields

$$\frac{p_2(X)}{p_Z(X)} \approx 1 + \frac{\langle a \rangle^2}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} l(x_i) l(x_k) \cos(\omega_2 t_i - \omega_2 t_k) + \frac{\sigma_a{}^2}{4} \left(\sum_{i=1}^{N} l'(x_i) + \sum_{i=1}^{N} [l(x_i)]^2 \right) + + \frac{\langle a \rangle^2}{4} \sum_{i=1}^{N} l'(x_i), \tag{12}$$

with a corresponding expression for $p_1(X)$, where $\sigma_a{}^2$ is the variance of a. We see that if $K = 1$, we obtain the same receiver as before [(11) and (12)], but if $K \neq 1$, then the receiver for the fast fading case is different from that for the slow fading case. The fast fading receiver involves $\langle a \rangle^2$ as well as the signal power (i.e., $\langle a^2 \rangle$), and is explicitly

$$G_2 + (1 - K) + (1 - K) \frac{\sigma_a{}^2}{4} \sum_{i=1}^{N} (l' + l^2) \ \underset{H_2}{\overset{\text{decide: } H_1}{\lessgtr}} \ KG_1,$$

$$G_j = \frac{\langle a \rangle^2}{2} \sum_i \sum_k l_i l_k \cos \omega_j (t_i - t_k), \qquad j = 1, 2. \tag{13}$$

Recently, Nirenberg [4] has shown, by quite different techniques, that the above results (Fig. 1) hold for m equiprobable incoherent signals.

B. LOBD for ON-OFF Incoherent Signals

The ON-OFF incoherent signaling case turns out to be a special situation. Consider the two hypotheses:

$$H_1: X(t) = Z(t), \qquad 0 \leq t < T$$

$$H_2: X(t) = Z(t) + S(t, \theta), \qquad 0 \leq t < T, \tag{14}$$

where $S(t, \theta) = \sqrt{2S} \cos(\omega t + \phi)$, and ϕ is uniformly distributed. Let us proceed ($K = 1$) as in (6) earlier, but now for the ON-OFF situation (14), using only second-order terms, $0(S)$. We see at once that, under H_1, neither H_1, the receiver structure, nor the threshold depend on the signal strength S. That is, under H_1, performance is independent of signal level. This is not too satisfying physically, since we may reasonably expect that for ON-OFF signals, the threshold, at least, should depend on signal size. [We can also readily see how this state

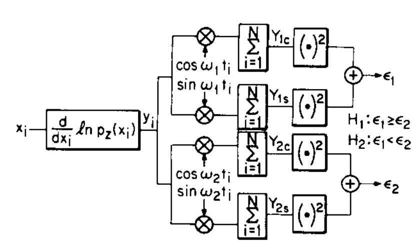

Figure 1. LOBD for binary NCFSK, constant or slow fading signal, $K = 1$, from eq. (11).

of affairs comes about by looking at the Gaussian case, where the optimum receiver is well known; cf. Sec. 5.1.2 [5].] Accordingly, for a physically meaningful threshold receiver for the incoherent ON-OFF case, we must include higher order terms, at least in the determination of the threshold. This is a special instance of insuring "consistency" (i.e., $P_e \to 0$ as $N \to \infty$, [6]).

Thus, we return to the case of impulsive interference, and find that the next nonzero terms in the threshold development of $p_2(X)/p_Z(X)$ are the 4th-order terms, namely

$$\frac{1}{4! p_Z(X)} \sum_{i_4=1}^{N} \sum_{i_3=1}^{N} \sum_{i_2=1}^{N} \sum_{i_1=1}^{N} \frac{\partial^4 p_Z(X)}{\partial x_{i_1} \partial x_{i_2} \partial x_{i_3} \partial x_{i_4}} \langle s_{i_1} s_{i_2} s_{i_3} s_{i_4} \rangle. \tag{15}$$

If we discard all the 4th-order terms except those which contribute to the threshold, that is, all terms except for $i_1 = i_2 = i_3 = i_4$, we obtain the (approximate) threshold

$$-\frac{1}{4!} \sum_{i=1}^{N} \frac{d^4 p_Z(x_i)}{dx_i{}^4} \langle s_i{}^4 \rangle = \frac{-3S^2}{2 \cdot 4!} \sum_{i=1}^{N} \frac{d^4 p_Z(x_i)}{dx_i{}^4}, \tag{16}$$

replacing the zero threshold of the incomplete, second-order development. Now the threshold depends on the signal level. [Whether using (16) insures the desired consistency must be determined from the Asymptotic Relative Efficiency (ARE), etc. [6].] The LOBD itself becomes here from (6), (9) and (16):

$$\left[\sum_i^N l(x_i) \cos \omega t_i \right]^2 + \left[\sum_i^N l(x_i) \sin \omega t_i \right]^2 \ \underset{H_2}{\overset{\text{decide: } H_1}{\lessgtr}} \ - \sum_i^N l'(x_i) - \frac{S}{8} \sum_i^N \frac{d^4 p_Z(x_i)}{dx_i{}^4}. \tag{17}$$

C. Threshold Receiver (LOBD) when Phase Estimation is Used

In the above, we have assumed that the (RF) phase is uniformly distributed $(-\pi, \pi)$. Improvement in performance can be gained, however, if we can learn what the RF phase may be. Thus, it is common in receiving systems to employ phase-tracking loops to obtain an estimate of the desired phase. We may term this device a "partially coherent receiver." When such phase estimation is used, it is common to use the Tikhonov distribution $p(\phi \mid \Omega)$ for the phase angle or phase error, viz:

$$p(\phi \mid \Omega) = \frac{\exp[\Omega \cos \phi]}{2\pi I_0(\Omega)}, \quad -\pi \leqslant \phi < \pi, \tag{18}$$

where Ω is a parameter which measures the spread of the PDF and which has an important physical significance in the study of phase estimation [Van Trees, [7], Chapter II.2]. For example, when, $\Omega = 0$, (18) yields as one extreme the previously treated cases uniform phase distribution and at the other limit, $\Omega \to \infty$, the coherent state of completely known phase is achieved.

Using this phase distribution in the threshold receiver development, with $s_{2i} = \sqrt{2S}\cos(\omega_2 t_i + \phi)$, we readily find that the 1st order terms (in S) in the LOBD are no longer zero, and are

$$\langle s_{2i} \rangle = \frac{I_1(\Omega)}{I_0(\Omega)} \sqrt{2S} \cos \omega_2 t_i. \tag{19}$$

[We note that $\langle s_{2i} \rangle = 0$ for $\Omega = 0$, as before (A, B above), and that $\langle s_{2i} \rangle = \sqrt{2S}\cos \omega_2 t_i$, as $\Omega \to \infty$, since $\lim_{\Omega \to \infty} (I_1(\Omega)/I_0(\Omega)) = 1$, which yields just the earlier result for the coherent case (Part I, Section V).] Then, limiting ourselves to 1st-order terms only, we see that the statistical test (LOBD + decision) reduces to

$$\sum_{i=1}^{N} l(x_i) \cos \omega_2 t_i \;\underset{H_2}{\overset{\underline{\text{decide:}}}{\overset{H_1}{\gtrless}}}\; \sum_{i=1}^{N} l(x_i) \cos \omega_1 t_i, \tag{20}$$

which is the same threshold receiver (LOBD) obtained previously in Section V, Part I for coherent signals (cf. Fig. 12, Part I, for coherent FSK in this case).

At this first level of approximation, the receiver is seen to be independent of the parameter Ω and thus makes no explicit use of the phase information ($\Omega \neq 0$) pertinent to detection. For this reason we may expect an improvement in performance (larger ARE) if knowledge of Ω is employed. This suggests that we include second order terms (S^2) in the LOBD as well.

Accordingly, for the 2nd order terms, we have

$$\langle s_{2i} s_{2j} \rangle = \int_{-\pi}^{\pi} 2S \cos(\omega_2 t_i + \phi) \cos(\omega_2 t_j + \phi) \cdot \frac{\exp[\Omega \cos \phi]}{2\pi I_0(\Omega)} \, d\phi$$

$$= S \cos(\omega_2 t_i - \omega_2 t_j) + \frac{S I_2(\Omega)}{I_0(\Omega)} \cos(\omega_2 t_i + \omega_2 t_j). \tag{21}$$

In our threshold (or LOBD) expansion, from (6) and (21), the 2nd order terms now give a sum of two double summations, the first consisting of slowly varying terms ($\omega_2 t_i - \omega_2 t_j$), the second of rapidly oscillating or "double frequency" terms ($\omega_2 t_i + \omega_2 t_j$), which largely cancel each other. Thus, the second double summation is negligible vis-a-vis the first [see, for example, Hancock and Wintz [8], Chapter 3]. With both the 1st order and 2nd order terms, the result is now a receiver which is the weighted sum of the purely coherent threshold receiver and the purely incoherent threshold receiver. The LOBD is shown in Figure 2 for $K = 1$, and is specifically

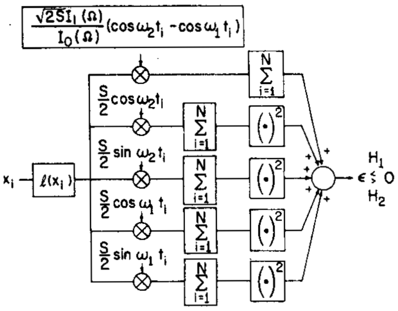

Figure 2. Second-order threshold receiver (LOBD) for NCFSK with phase estimation (22).

$$-\sqrt{2S}\frac{I_1(\Omega)}{I_0(\Omega)} \sum_i l(x_i)(\cos \omega_2 t_i - \cos \omega_1 t_i)$$

$$+ \frac{S}{2} \left\{ \left[\sum_i l(x_i) \cos \omega_2 t_i \right]^2 + \left[\sum_i l(x_i) \sin \omega_2 t_i \right]^2 - \left[\sum_i l(x_i) \cos \omega_1 t_i \right]^2 - \left[\sum_i l(x_i) \sin \omega_1 t_i \right]^2 \right.$$

$$\underline{\text{decide:}}$$

$$= \epsilon \;\underset{H_2}{\overset{H_1}{\lessgtr}}\; 0. \tag{22}$$

In Figure 2 we note that for $\Omega = 0$, the receiver reduces to that obtained earlier (Fig. 1) for uniformly distributed phase and as $\Omega \to \infty$; i.e., phase fully known at the receiver, we obtain the coherent LOBD derived previously (Part I) (but now with the addition of the 2nd-order terms, which improves the ARE). Finally, we remark that in studying optimum reception of coded, multiple-frequency keying with random variable phases, Nesenbergs [9] has shown (for Gaussian noise) that the appropriate receiver is also a weighted sum of the standard linear (coherent) receiver and the standard quadrature (incoherent) receiver. We have shown here that this is a completely canonical result: one always gets this type of LOBD receiver for partially known phase, regardless of the explicit structure of the pdf of the interference.

III. OPTIMUM DETECTION OF GENERAL INCOHERENT SIGNALS

In the previous Section we obtained threshold or locally optimum receivers for a variety of binary incoherent and partially coherent reception situations. Here we examine the general question of composite hypothesis testing; i.e., without any small-signal assumptions. In general, we are not able to obtain any explicit receiver structures, but we are able to obtain expressions useful in determining a bound on optimum performance. The techniques developed here are helpful in any composite hypothesis testing problem, such as the previous incoherent examples (Sec. II). The technical difficulties lie in performing the averages over the random parameter (vector) θ in the likelihood ratio, cf. (4), $\theta = (a, \phi)$.

Accordingly, let us look at Class A interference specifically in our effort to establish explicit performance bounds, in contrast to Sections I and II above where the analysis gave LOBD's which were canonical vis-a-vis interference and signal waveforms.

Our pdf for this interference is

$$p_Z(z) = \sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m!\sqrt{2\pi\sigma_m^2}} e^{-z^2/2\sigma_m^2}, \tag{23}$$

where $\sigma_m^2 = (m/A + \Gamma')/(1 + \Gamma')$. Each term of the sum (23) can be expressed as a Fourier transform, so that, with a change of integration and carrying out the summation, with the help of the characteristic function $F_1(j\xi)_Z$ [1], we get

$$p_Z(z) = \frac{e^{-A}}{2\pi} \int_{-\infty}^{\infty} e^{j\xi z} e^{-\frac{\xi^2}{2}\left(\frac{\Gamma'}{1+\Gamma'}\right)} \exp\left[A e^{-\frac{\xi^2}{2A(1+\Gamma')}}\right] d\xi. \tag{24}$$

The Nth-order distribution of $\{X_i\}$, with N-independent samples, can therefore be written as an N-fold integral of which (24) is a typical factor, $p_Z(x_i)$. Turning next to the incoherent case (3) in which the phase is uniformly distributed, we have

$$p_2(X) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{e^{-NA}}{(2\pi)^N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{j\sum_{i=1}^{N} \xi_i x_i}$$

$$\times e^{-j\sum_{i=1}^{N} \xi_i \sqrt{2S}\cos(\omega_2 t_i + \phi)} \times e^{-\frac{\Gamma'}{2(1+\Gamma')}\sum_{i=1}^{N}\xi_i^2}$$

$$\times \exp\left[A \sum_{i=1}^{N} e^{-\frac{\xi_i^2}{2A(1+\Gamma')}}\right] d\xi_1 \cdots d\xi_N \, d\phi. \tag{25}$$

The required average (over ϕ) in (25), denoted by I, is then

$$I = J_0(\sqrt{L_c^2 + L_S^2}), \tag{26}$$

where

$$L_c = \sum_{i=1}^{N} \xi_i \sqrt{2S} \cos \omega_2 t_i,$$

$$L_s = \sum_{i=1}^{N} \xi_i \sqrt{2S} \sin \omega_2 t_i. \tag{26a}$$

[We now remark that if the small-signal approximation $J_0(x) \approx 1 - (x^2/4)$ is used, we obtain results identical to (6) for incoherent threshold reception [Section II], cf. Sec. 5.2, [5]].

Rather than using the weak-signal approximation to $J_0(x) = 1 - x^2/4$, we can use the steepest-descent approximation

$$J_0(x) \approx e^{-x^2/4}. \tag{27}$$

This yields correct limiting behavior for both large and small values of signal amplitude [see [1], p. 27 et seq.]. Now for the argument $L_c^2 + L_s^2$ in (26), we have

$$L_c^2 + L_s^2 = \sum_{k=1}^{N} \sum_{i=1}^{N} \xi_i \xi_k 2S \cos \omega_2 (t_i - t_k) \tag{28a}$$

$$\cong 2S \sum_i \xi_i^2 \tag{28b}$$

where we can neglect the summation of oscillating terms $(i \neq k)$ when N is large, so that these are essentially cancelling oscillations in the detection period $[0, T]$, in total, negligible compared to the $(i = k)$ contribution. Our making this assumption destroys insight into receiver structure, but is useful in calculating performance. Applying (28b) to (26) in (25), we get (the incoherent result)

$$p_2(X) = \frac{e^{-NA}}{(2\pi)^N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{j\sum_{i=1}^{N} \xi_i x_i - \frac{S}{2}\sum_{i=1}^{N} \xi_i^2}$$

$$\times e^{-\frac{\Gamma'}{2(1+\Gamma')}\sum_{i=1}^{N}\xi_i^2} \exp\left[A \sum_{i=1}^{N} e^{-\frac{\xi_i^2}{2A(1+\Gamma')}}\right] d\xi_1 \cdots d\xi_N \tag{29a}$$

$$p_2(X) = \prod_{i=1}^{N} \sum_{m=0}^{N} e^{-A} \frac{A^m}{m!} \frac{1}{\sqrt{2\pi(\sigma_m^2 + S)}} e^{-x_i^2/2(\sigma_m^2+S)}. \tag{29b}$$

[Note that $p_2(X)$ is a proper pdf; i.e., it is everywhere ($\geqslant 0$) and integrates on $(-\infty, \infty)$ to unity.] Using (29b), we find finally that the likelihood ratio and test for the ON-OFF incoherent case in Class A noise now becomes approximately, for small and large signal levels,

$$\Lambda(X) \cong \frac{\prod_{i=1}^{N} \sum_{m=0}^{\infty} e^{-A} \frac{1}{\sqrt{2\pi(\sigma_m^2 + S)}} e^{-x_i^2/2(\sigma_m^2+S)}}{\prod_{i=1}^{N} \sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m!} \frac{1}{\sqrt{2\pi\sigma_m^2}} e^{-x_i^2/2\sigma_m^2}} \quad \underset{H_2}{\overset{H_1}{\gtrless}} K. \tag{30}$$

decide:

[We shall use (30) in the next Section (IV, B) to compute performance for this ON-OFF incoherent case (with $K = 1$).]

IV. DETERMINATION OF INCOHERENT PERFORMANCE

Having obtained LOBD receiver structures for incoherent reception under arbitrary classes (pdf's) of interference, we wish now to evaluate the performance of these receivers. Results for performance of these LOBD structures will be obtained in Section [A]. We than apply these results to our case of Class A interference and compare receiver performance with that calculated in Part I for coherent LOBD receivers to examine the degradation which occurs when phase information is lacking. In Sec. II we obtained an approximation to the likelihood ratio (30) for the important case of optimum incoherent ON-OFF reception, at all signal levels. We will accordingly use (30) to determine the corresponding receiver performance and to compare this performance with that of the corresponding coherent system in Section [B]. Finally, the performance of suboptimum incoherent correlation receivers is also computed for our Class A interference in Section [C].

A. Performance of Threshold (LOBD) Receivers for Binary NCFSK

In this section we use the receiver structures and results obtained in Sections II and III above to evaluate the performance of the canonical incoherent receiver. We start with the detection situation

$$\left.\begin{aligned} H_1: X(t) &= Z(t) + S_1(t, \phi), \qquad 0 \leqslant t < T \\ H_2: X(t) &= Z(t) + S_2(t, \phi), \qquad 0 \leqslant t < T \end{aligned}\right\} \tag{31}$$

where in the special case of Binary NCFSK signals,

$$\begin{aligned} S_1(t, \phi) &= \sqrt{2S} \cos (\omega_1 t + \phi) \\ S_2(t, \phi) &= \sqrt{2S} \cos (\omega_2 t + \phi), \end{aligned} \tag{31a}$$

and the phase ϕ is uniformly distributed. For the threshold choice $K = 1$, the appropriate LOBD receiver is shown in Figure 1 for these particular signals.

As was discussed in Part I, (Section V), the action of the nonlinearity $l(x)$ allows us to obtain an estimate of performance via the Central Limit Theorem. That is, we note that by the CLT (for these independent samples), Y_{1c}, Y_{1s}, Y_{2c}, and Y_{2s} in Figure 1 are (asymptotically) normally distributed. We shall first carry out an "exact" evaluation, up to the formal representation of the probability of error, cf. (38), which, however, requires numerical evaluation. Then, we shall simplify these results, again under the LOBD conditions to obtain the desired approximation explicitly, without having to go the route of numerical calculations.

Let us suppose H_1 to be true, so that $x_i = z_i + s_{1i}$, $s_{1i} = \sqrt{2S} \cos (\omega_1 t_i + \phi)$, and $P_e \mid H_1 = \text{Prob}\ [\epsilon_1 < \epsilon_2]$. The squaring and summation operations in the receiver (Fig. 1) yield receiver operation independent of the unknown phase. We therefore can set $\phi = 0$. We now use the results of Part I, Eqs. (36b) and (38b), to calculate the mean and variance of y_i under H_1, which for these particular signals (31a) become (for the top branch of the receiver)

$$\begin{aligned} E[Y_{1c} \mid H_1] &\cong -L \sum_{i=1}^{N} s_{1i} \cos \omega_1 t_i \\ &= -L\sqrt{2S} \sum_{i=1}^{N} \cos^2 \omega_1 t_i, \end{aligned} \tag{32a}$$

and

$$\operatorname{Var}\ [Y_{1c} \mid H_1] = \sum_{i=1}^{N} (L - 2SL^2 \cos^2 \omega_1 t_i) \cos^2 \omega t_i. \tag{32b}$$

For Y_{1s}, we get

$$\begin{aligned} E[Y_{1s} \mid H_1] &= -L\sqrt{2S} \sum_{i=1}^{N} \cos \omega_1 t_1 \sin \omega_1 t_i \\ &= -\frac{L\sqrt{2S}}{2} \sum_{i=1}^{N} \sin 2\omega_1 t_i \approx 0, \end{aligned} \tag{33a}$$

and

$$\begin{aligned} \operatorname{Var}\ [Y_{1s} \mid H_1] &= \sum_{i=1}^{N} (L - 2SL^2 \cos^2 \omega_1 t_i) \sin^2 \omega_1 t_i \\ &= \sum_{i=1}^{N} \quad L \sin^2 \omega_1 t_i - \frac{SL^2}{2} \sin^2 2\omega_1 t_i \quad . \end{aligned} \tag{33b}$$

For the lower, (ω_2) branch of the receiver, we obtain similarly:

$$E[Y_{2c} \mid H_1] = -L\sqrt{2S} \sum_{i=1}^{N} \cos \omega_1 t_i \cos \omega_2 t_i \approx 0, \tag{34a}$$

$$\operatorname{Var}\ [Y_{2c} \mid H_1] = \sum_{i=1}^{N} (L - 2SL^2 \cos^2 \omega_1 t_i) \cos^2 \omega_2 t_i, \tag{34b}$$

$$E[Y_{2s} \mid H_1] = -L\sqrt{2S} \sum_{i=1}^{N} \cos \omega_1 t_i \sin \omega_2 t_i \approx 0, \tag{35a}$$

and

$$\operatorname{Var}\ [Y_{2s} \mid H_1] = \sum_{i=1}^{N} (L - 2SL^2 \cos^2 \omega_1 t_i) \sin^2 \omega_2 t_i. \tag{35b}$$

Now we have (cf. Fig. 1) $\epsilon_1 \equiv Y_{1c}^2 + Y_{1s}^2$, so that (Omura and Kailath [10], p. 69) the pdf of the LOBD under H_1 is explicitly

$$p_1(\epsilon_1) = \frac{1}{2\sigma_{1c}\sigma_{1s}} \exp\left(-\frac{\epsilon_1 + \mu_{1c}^2}{2\sigma_{1c}^2}\right) \sum_{j=0}^{\infty} \frac{\Gamma\left(\frac{1}{2} + j\right)}{j!\,\Gamma\left(\frac{1}{2}\right)} \cdot \left[\frac{\sqrt{\epsilon_1}(\sigma_{1s}^2 - \sigma_{1c}^2)}{\mu_{1c}\sigma_{1s}^2}\right]^j I_j\left(\frac{\sqrt{\epsilon_1}\,|\mu_{1c}|}{\sigma_{1c}^2}\right), \tag{36}$$

where

$$\mu_{1c} = E[Y_{1c} \mid H_1], \quad \sigma_{1c}^2 = \text{Var}\,[Y_{1c} \mid H_1],$$
$$\sigma_{1s}^2 = \text{Var}\,[Y_{1s} \mid H_1]. \tag{36a}$$

Also, for $\epsilon_2 \equiv Y_{2c}^2 + Y_{2s}^2$ (cf. Fig. 1) we have similarly [10], p. 62,

$$p_2(\epsilon_2) = \frac{1}{2\sigma_{2c}\sigma_{2s}} \exp\left[-\frac{(\sigma_{2c}^2 + \sigma_{2s}^2)\,\epsilon_2}{4\sigma_{2c}^2\sigma_{2s}^2}\right] \cdot I_0\left(\frac{|\sigma_{2c}^2 - \sigma_{2s}^2|\,\epsilon_2}{4\sigma_{2c}^2\sigma_{2s}^2}\right), \tag{37}$$

where

$$\sigma_{2c}^2 = \text{Var}\,[Y_{2c} \mid H_1], \quad \sigma_{2s}^2 = \text{Var}\,[Y_{2s} \mid H_1]. \tag{37a}$$

For H_2 true, we obtain completely symmetrical results, so the probability of error is given by

$$P_e = P_e \mid H_1 = \text{Prob}\,[\epsilon_1 < \epsilon_2] = \int_0^\infty p_2(\epsilon_2) \int_0^{\epsilon_2} p_1(\epsilon_1)\, d\epsilon_1\, d\epsilon_2. \tag{38}$$

In (38), the integral over ϵ_1 is simply the distribution function of ϵ_1, evaluated at ϵ_2. This is a known result that gives an infinite series of Q_M-functions (see Omura and Kailath, [10], p. 22). The remaining integration, involving products of exponentials, Q_M-functions, and modified Bessel functions, I_0, can also be evaluated analytically (Nuttall, [11], [13]). These rather complex results then require numerical evaluation.

However, we can take advantage of the small-signal (LOBD) assumption and relations, and avoid the necessity of numerical evaluations. We make the following observations: From (32b), we have

$$\sigma_{1c}^2 = L \sum_{i=1}^{N} \cos^2 \omega_1 t_i - 2SL \sum_{i=1}^{N} \cos^4 \omega_1 t_i \approx \frac{LN}{2}\left(1 - \frac{3SL}{2}\right). \tag{39a}$$

Likewise, from (33b) we get

$$\sigma_{1s}^2 \approx \frac{LN}{2}\left(1 - \frac{SL}{2}\right). \tag{39b}$$

Now both L and N are large, and from our small signal assumption $SL \ll 1$, so that

$$\sigma_{1c}^2 \approx \sigma_{1s}^2 \approx \frac{NL}{2} \equiv \sigma_1^2, \tag{40}$$

and, therefore, (36) reduces (approximately) to

$$p_1(\epsilon_1) \approx \frac{1}{2\sigma_1^2} \exp\left(-\frac{\epsilon_1 + \mu_{1c}^2}{2\sigma_1^2}\right) I_0\left(\frac{\sqrt{\epsilon_1}\,\mu_{1c}}{\sigma_1^2}\right). \tag{41}$$

Similarly, from (34b), (35b) we get

$$\sigma_{2c}^2 \approx \frac{LN}{2}(1 - SL) \approx \sigma_{2s}^2 \equiv \sigma_2^2. \tag{42}$$

Therefore, we have for (37)

$$p_2(\epsilon_2) \approx \frac{1}{2\sigma_2^2} \exp\left(-\frac{\epsilon_2}{2\sigma_2^2}\right). \tag{43}$$

Applying (41) and (43) to (38), we obtain

$$p_e \approx \int_0^\infty \frac{1}{2\sigma_2^2} \exp\left(-\frac{\epsilon_2}{2\sigma_2^2}\right) \left\{1 - Q\left(\frac{\mu_{1c}}{\sigma_1}, \frac{\sqrt{\epsilon_2}}{\sigma_1}\right)\right\} d\epsilon_2 \tag{44}$$

where

$$Q(a, b) \equiv \int_b^\infty x \exp\left(-\frac{x^2 + a^2}{2}\right) I_0(ax)\, dx. \tag{44a}$$

Next, we evaluate (44) using (8) or (13) of Nutall [11], to get

$$P_e \approx \frac{1}{1 + \sigma_1^2/\sigma_2^2} \exp\left[-\frac{\mu_{1c}^2}{2(\sigma_1^2 + \sigma_2^2)}\right]. \tag{45}$$

Since $SL \ll 1$, we can use $\sigma_1^2 \approx \sigma_2^2 \approx NL/2$, write (32a) as

$$\mu_{1c} \approx \frac{NL}{2}\sqrt{2S}, \tag{46}$$

to obtain finally the simple (approximate) result

$$P_e \approx \frac{1}{2} \exp\left[-\frac{NLS}{4}\right], \quad (SL \ll 1). \tag{47}$$

Our result (47) gives the desired estimate of performance for the optimum incoherent threshold receiver for the specific (NCFSK) signals (31a). In Part I we obtained corresponding results for the coherent case [Section V]. The result (51), Part I, applies for coherent antipodal signals (CPSK); i.e., $\phi = -1$, but is easily modified for CFSK signals ($\phi = 0$). Figure 3 shows the NCFSK performance [from (47)] and the corresponding performance for CFSK for our case $A = 0.35$, $\Gamma' = 0.5 \times 10^{-3}$ (i.e., $L = 1340$). As expected, incoherent LOBD performance is degraded vis-à-vis the corresponding coherent operation, but all performance greatly improves with N ($\approx$ time-bandwidth product).

B. Performance of the Optimum ON-OFF Incoherent Receiver in Class A Noise

We turn now to the result (30) obtained in Section III for the likelihood ratio and decision process in the case of the optimum ON-OFF incoherent receiver for all signal levels in Class A noise. Using the techniques [Section III] developed

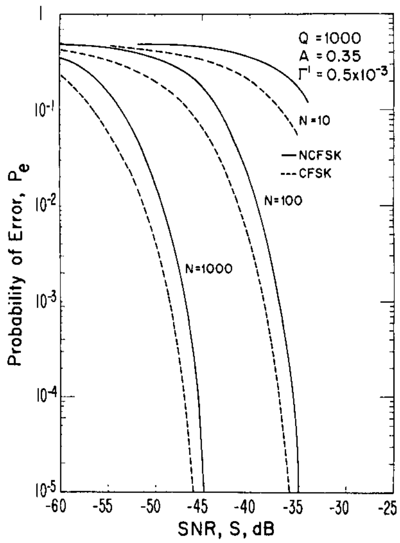

Figure 3. Performance of threshold receiver for binary NCFSK [from (47)] and for binary CFSK [from Part 1, (42)] for the Class A interference case $A = 0.35$ and $\Gamma' = 0.5 \times 10^{-3}$.

in Part I, we obtain the bound on performance for $K = 1$, $(q_1 = q_1 = 1/2)$, (18), Part I, specifically here

$$P_e \leqslant \frac{1}{2} \prod_{i=1}^{N} \int_{-\infty}^{\infty} \left[\sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m! \sqrt{2\pi(\sigma_m{}^2 + S)}} \cdot e^{-x_i{}^2/2(\sigma_m{}^2+S)} \right]^{1-\alpha} \times \left[\sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m! \sqrt{2\pi \sigma_m{}^2}} \cdot e^{-x_i{}^2/2\sigma_m{}^2} \right]^{\alpha} dx, \tag{48}$$

cf. (19b), Part I. Since now all signal samples are the same, we have (cf. (21a), Part I)

$$P_e{}^* \leqslant \frac{1}{2} [I_{\alpha^*}(S)]^N, \tag{49}$$

where now

$$I_{\alpha^*}(S) = 2 \int_0^{\infty} \left[\sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m! \sqrt{2\pi(\sigma_m{}^2 + S)}} \cdot e^{-x^2/2(\sigma_m{}^2+S)} \right]^{1-\alpha^*} \times \left[\sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m! \sqrt{2\pi \sigma_m{}^2}} \cdot e^{-x^2/2\sigma_m{}^2} \right]^{\alpha^*} dx, \tag{50}$$

and α^* is the value of α that minimizes the right-hand side of (50). The proper values of α^* depends on signal level S, so that computer evaluation also involves finding α^* for each S. Table 1 gives α^* for various values of signal power S, obtained by computer search.

Figure 4 shows the estimate $P_e{}^*$ of performance for $N = 10$ from (49). Also shown is the performance bound $\hat{P}_e$ obtained

TABLE 1
THE OPTIMIZING VALUE OF α^* FOR VARIOUS SIGNAL POWER LEVELS S

S(dB)	α^*	S(dB)	α^*	S(dB)	α^*
30	.30	0	.57	-30	.45
25	.38	- 5	.54	-35	.47
20	.46	-10	.50	-40	.485
15	.53	-15	.47	-45	.49
10	.58	-20	.44	-50	.495
5	.59	-25	.43		

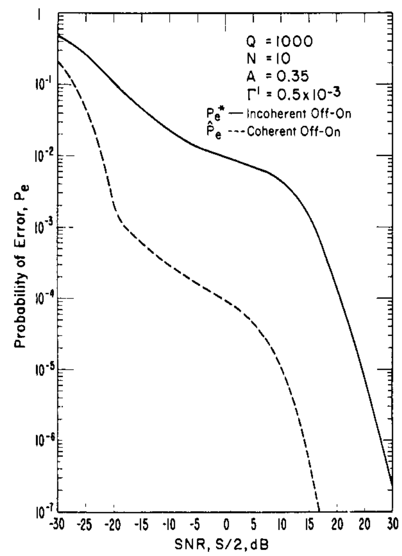

Figure 4. Performance estimate for the optimum incoherent ON-OFF receiver [from (49)] and the performance bound [(34), Part I] for the optimum coherent ON-OFF receiver for the interference case $A = 0.35$, $\Gamma' = 0.5 \times 10^{-3}$ and for $N = 10$.

in Part I for coherent ON-OFF signaling. Figure 5 shows the performance bound $\hat{P}_e$ and performance estimate $P_e{}^*$ (coherent and incoherent) for $N = 10$, 100, and 1000. On these Figures the SNR is given by $S/2$; i.e., the average signal power in the two signals, one signal being zero. The results indicate that, not unexpectedly, the incoherent optimum receiver performs substantially worse than the coherent optimum detector. However, our estimate $P_e{}^*$ of performance for the incoherent case is quite likely much cruder than the coherent bound $\hat{P}_e$, especially for small N.

C. Performance of Noncoherent Correlation (i.e., Suboptimum) Receivers in Class A Interference

It remains now to compute the performance of the current suboptimum correlation receivers in Class A "impulsive" interference. The performance for binary NCFSK is quite easy to obtain. For arbitrary interference Montgomery [12] has shown that the probability of error is given by

$$P_e = \frac{1}{2} \text{Prob [rms noise envelope} > \text{rms signal amplitude]}. \tag{51a}$$

In our Class A interference, this gives us [from (9) of Part I],

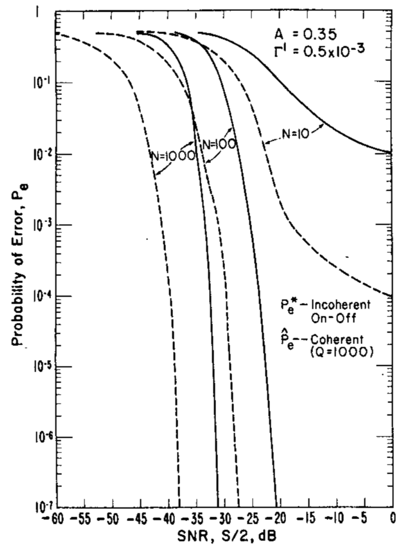

Figure 5. Performance estimate for the optimum incoherent ON-OFF receiver [from (49)] and the performance bound for the optimum coherent ON-OFF receiver [(34), Part I] for the interference case $A = 0.35$, $\Gamma' = 0.5 \times 10^{-3}$ and for $N = 10, 100$, and 1000.

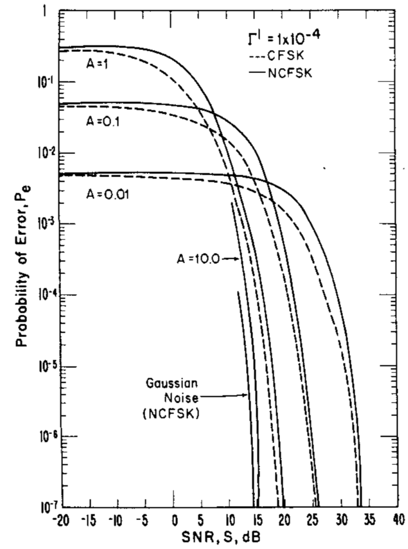

Figure 6. Performance of NCFSK correlation receiver [from (51b)] and CFSK correlation receiver [from (41), Part 1] in Class A interference for $\Gamma' = 1 \times 10^{-4}$ and $A = 0.01, 0.1, 1$, and 10.

$$P_e = \frac{1}{2} e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} e^{-S/2\sigma_m^2}, \tag{51b}$$

where, as before, S is the signal power (and also the SNR because of our normalization).

Figure 6 shows the performance of the standard (i.e., optimum for Gaussian noise NCFSK receiver, from (51b), for $\Gamma' = 1 \times 10^{-4}$ and $A = 0.01, 0.1, 1$, and 10. Also shown is the standard CFSK performance from (31) of Part I, $k = 2$. As expected, for the standard receivers the use of NCFSK results only in a small degradation of performance compared to CFSK for large S (small P_e).

The performance of the suboptimum incoherent ON-OFF system is more difficult to determine. The situation is sketched in Figure 7. In Figure 7 the probability of error is given by

$$P_e = \frac{1}{2} [\text{Area I} + \text{Area II}], \tag{52}$$

where the threshold K depends on signal level S and is set to minimize P_e. From (11a) of Part I we have the pdf under H_1 (noise level), from (12), Part I, the pdf of the envelope of signal plus noise. Therefore, it follows directly from these relations in (52) that

$$\text{Area I} = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} e^{-K^2/\sigma_m^2}, \tag{53a}$$

and

$$\text{Area II} = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \left(\frac{2}{\sigma_m^2}\right) e^{-2S/\sigma_m^2} \cdot \int_0^K \mathcal{E} e^{-\mathcal{E}^2/\sigma_m^2} I_0\left(\frac{2\mathcal{E}\sqrt{2S}}{\sigma_m^2}\right) d\mathcal{E}, \tag{53b}$$

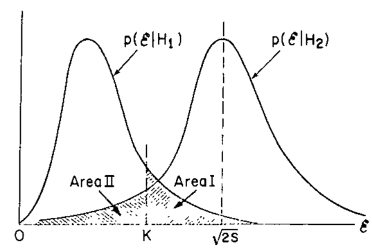

Figure 7. Probability of error for incoherent ON-OFF (suboptimum) correlation receiver.

so that [11], [13]

$$P_e = \frac{1}{2} \left\{ 1 + e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \left[e^{-K^2/\sigma_m^2} - Q\left(\frac{2\sqrt{S}}{\sigma_m}, \frac{K\sqrt{2}}{\sigma_m}\right) \right] \right\}. \tag{54}$$

For the performance of the suboptimum ON-OFF system numerical evaluation of (54) is required. However, we can approximately compare performance of the standard suboptimum incoherent ON-OFF receiver with that of our optimum receiver and avoid additional numerical evaluation by comparing the performance of the *coherent* ON-OFF correlation receiver with out optimum *incoherent* ON-OFF receiver. This is a reasonable approximation, since we have seen that for the normal correlation receivers in Class A interference, there is little difference between coherent and incoherent performance for small P_e (e.g., Fig. 6). Accordingly, Figure 8 shows (for $N = 100$) the performance of the coherent ON-OFF correlation receiver from (41) of Part I vis-à-vis the estimated performance of the optimum incoherent receiver from Section IVB above (cf. Figs. 5, 6) for our example $\Gamma' = 0.5 \times 10^{-3}$ and $A = 0.35$. As in the coherent cases, these results indicate that substantial improvement (25 dB or more) over conventional correlation receivers can be achieved.

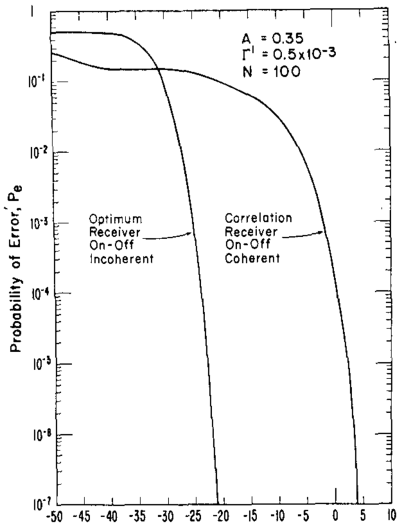

Figure 8. Performance of the suboptimum coherent ON-OFF correlation receiver [from (41), Part I] and the estimated performance of the optimum incoherent ON-OFF receiver [from Sec. IVB] for the Class A interference case $A = 0.35$, $\Gamma' = 0.5 \times 10^{-3}$, for $N = 100$.

TABLE 2
SUMMARY OF PRINCIPAL NEW RESULTS

Class A Interference [1],[2]		
Signaling Situation	Detector Structure	Performance
Coherent (θ known) $H_1: X(t)=S_1(t)+Z(t)$, $H_2: X(t)=S_2(t)+Z(t)$ Antipodal, orthogonal and ON-OFF	Likelihood ratio, (Part I, Sec. III)	Performance bound, $\hat{P}_e$ (Part I, Sec. III)
Antipodal, orthogonal and ON-OFF	LOBD (Part I, Sec. I)	Performance estimate P_e (Part I, Section V)
	Correlation receiver (optimum in gauss)	"Exact" P_e (Part I, Sec. IV)
Incoherent (θ unknown) $H_1: X(t)=S_1(t;\theta)+Z(t)$, $H_2: X(t)=S_2(t;\theta)+Z(t)$ NCFSK, $\theta = \{a, \phi\}$ K arbitrary, ϕ: uniform		
a; constant	LOBD (Sec. IIA)	Performance estimate, ($K = 1$), P_e (Sec. IVA)
a; slow-fading	LOBD (Sec. IIA)	
a; fast-fading	LOBD (Sec. IIA)	
NCFSK with phase estimation	1st and 2nd order LOBD's (Sec. IIC)	
NCFSK	Correlation receiver	"Exact" P_e (Sec. IVC)
ON-OFF	LOBD (Sec. IIB) Approximation of $\Lambda(\underline{X})$ (Sec. III)	Performance estimate, P_e (Sec. IVB)
	Correlation receiver	"Exact" P_e (Sec. IVC)

V. SUMMARY AND CONCLUSIONS

Since communication systems are seldom interfered with by classical white Gaussian noise, it has been the object of these papers (Parts I, II) to apply a recently developed statistical-physical, canonical model [1], [2] of generalized impulsive interference to real-world problems of signal detection. The pertinent features of this impulsive interference model are summarized in Part I, including typical excellent agreement with a variety of measured statistics. The critical feature of all EM interference of these common classes is the highly non-Gaussian character of the interference.

Among the principal new results are canonical optimum detection algorithms for coherent and incoherent binary detection, also specialized to the three basic waveforms used in digital signaling; e.g., antipodal, orthogonal, and ON-OFF keying. Other important results are performance bounds for these signaling situations. Since it is known that in order to gain significant improvement over current receivers, the number of independent samples of the received interference waveform must be large, performance measures are developed which are parametric in the number of samples, or equivalently, in the time-bandwidth product. Performance measures of current suboptimum receivers (e.g., the conventional matched-filter detectors for Gaussian noise) are also obtained and compared to the performance of the corresponding optimum systems for these highly non-Gaussian EM environments. It is shown that substantial savings in signal power and/or in spectrum space can also be achieved.

Since physical realization of totally optimum detection algorithms cannot, in general, be economically obtained, the corresponding locally optimum or Bayes threshold (LOBD) receivers are derived and their performance specified. These threshold receiver structures are canonical, in that the *form* of their structure is independent of the explicit nature of the interference and signal waveform. Locally optimum structures are also obtained for coherent and incoherent detection subject to various kinds of fading signals. The important threshold case in which phase estimation is used (so-called "partially coherent detection") is also treated.

It is convenient to summarize the particular detection problems examined and the results obtained here and in Part I in Table 2. As noted previously, all the performance results, including the performance of correlation receivers in Class A interference, are new. Many of the LOBD receiver structures are believed also to be new. Specifically, it is shown that the LOBD for NCFSK with slow-fading signals is different from the LOBD with fast-fading signals when the threshold $K \neq 1$. On the other hand, for $K = 1$, the LOBD structure has been known for some time and is the same (cf. Fig. 1) for constant slow-fading or fast-fading signals. [This result has recently been demonstrated for m-level NCFSK by Nirenberg [4], who also shows that if the signals contain a desired amplitude modulation, additional nonlinear processing is required in the LOBD structure. However, the techniques used by Nirenberg to obtain these results are quite different from those used in this study.] If $K \neq 1$, the LOBD structure for slow-fading signals also involves the second moment of the fading distribution (i.e., the signal power), whereas for fast-fading signals, the LOBD structure involves the mean of the signal fading distribution as well (cf. (12)). In addition, the LOBD for ON-OFF incoherent signaling is shown to be a special case, requiring the inclusion of higher order terms in the expansion of the likelihood ratio to insure consistency (Sec. IIB above). Furthermore, the LOBD structure for NCFSK with phase estimation is found to be canonical (as are the other LOBD structures) and to involve a weighted linear combination of the coherent and incoherent LOBD's (e.g., Fig. 2).

Although an upper bound on the performance of the completely optimum receiver for coherent signals is obtained in Part I, correspondingly general results for incoherent signals are much more difficult to achieve. For incoherent operation, the only performance result obtained here for the completely optimum receiver is the performance estimate for incoherent ON-OFF signals in Class A interference (Sec. IVB). [We remark that here and in Part I, all calculations of performance are performed with the pdf of the interference in normalized form, so that the noise power is unity and the SNR is given by the signal power S.] In actual situations, performance of the optimum and locally optimum receivers is not a function of the SNR alone because of the required nonlinear processing. That is; for actual problems of interest, the performance algorithms must use the pdf of the interference in unnormalized or absolute form. Accordingly, performance in general depends on the absolute interference *level* as well as on the SNR. In this sense, then, these receivers are *adaptive*; i.e., they must be able to adjust to the parameters (level, etc.) of the interference.

Various problems remain for further investigation (some now underway by the authors). Among the more immediate is the extension of the results to include Class B and Class C (= Class A + Class B) interference [1], [2]. Others are the following: Important digital signaling situations, such as minimal shift keying and differential phase shift keying, and the evaluation of the performance of some of the LOBD structures which have been derived here, but not explicitly determined (e.g., NCFSK with phase estimation). Some important characteristics of the LOBD's, such as the ARE (asymptotic relative efficiency) require more quantitative attention [cf. Section V, Part I]. For example, how "small" is a small signal in the threshold receiver development? Since the truly optimum and locally optimum receivers must be adaptive, techniques by which an actual receiver can estimate the required, changing interference parameters need to be developed. Multiplicative interference, such as frequency selective fading, also needs to be considered, for various real world communications channels, especially when we attempt to specify performance in terms of a required time-bandwidth product, since this type of interference usually restricts the amount of bandwidth which can be effectively used. Finally, we emphasize again the realistic interfering properties of the EM environment, man-made and natural, which our systems must work against, properties which are characterized by highly non-Gaussian statistics [1, 2].

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